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Axially Symmetric Cosmological Micro Model in Barber's Modified Theory of Einstein General Relativity

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Abstract Axially symmetric Bianchi type-I cosmological micro model is obtained in Barber's (Gen. Relativ. Gravit. 14:117, 1982) modified theory of general relativity. Some properties of the model are discussed.

Keywords Anisotropy · Barber's modified theory of relativity (self creation theory) · Micro model · Bianchi type-I space time

1 Introduction

Barber has invented two continuous self creation theories by modifying the Brans and Dicke [2] theory and general relativity. These modified theories create the universe out of self-contained gravitational scalar and matter fields. Brans has pointed out that Barber's first theory is not only in agreement with experiment but also inconsistent in general. Barber's second theory is a modification of general relativity to a variable G-theory. In this theory the scalar field does not directly gravitate but simply divides the matter tensor, acting as a reciprocal gravitational constant. Singh and Deo [3], Reddy [4–6], Maharaj and Beesham [7], Venkateswarlu and Reddy [8–10], Shanti and Rao [11], Mohanty et al. [12, 13] are some of the authors who have investigated various aspects of Barber's self creation theories. Mohanty et al. [14] have obtained a micro and macro cosmological models in the presence of massless scalar field interacted with perfect fluid. Panigrahi and Sahu [15–17] have also obtained plane symmetric inhomogeneous cosmological micro and macro models and plane symmetric mesonic stiff fluid models in Barber's second theory of gravitation.

In this paper, we consider an axially symmetric Bianchi type-I metric in Barber's self-creation theory of gravitation with massless scalar field distribution and constructed a anisotropic homogeneous axially symmetric Bianchi type-I cosmological micro model. Some physical and kinematical properties of the cosmological model are also discussed.

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2 Field Equations and Micro Model

We consider an axially symmetric and spatially homogeneous Bianchi-I metric given by

$$ds^2 = -dt^2 + \exp(2\alpha)dx^2 + \exp(2\beta)(dy^2 + dz^2), \tag{1}$$

where $\alpha = \alpha(t)$ and $\beta = \beta(t)$.

The Einstein-Barber micro field equations in second self creation theory are

$$G_{ij} \equiv R_{ij} - \frac{1}{2}g_{ij}R = -8\pi\phi^{-1}T_{ij} \quad \text{and} \tag{2}$$

$$\Box\phi = \frac{8\pi}{3}\lambda T, \tag{3}$$

where 'ϕ' is the Barber's scalar, T_{ij} is the energy momentum tensor for massless scalar field, $\Box\phi$ is the invariant D'Alembertian, T is the trace of energy momentum tensor T_{ij}. λ is a coupling constant to be determined from experiment and $0 < |\lambda| < 1/10$.

In the limit $\lambda \to 0$, this theory approaches the Einstein's theory in every respect. Due to the nature of the space time Barber's scalar ϕ is a function of 't'.

In order to study the cosmological effects in microscopic level the energy-momentum tensor T_{ij} [3] for a micro matter scalar field representing massless scalar field distribution is given by

$$T_{ij} = v_i v_j - \frac{1}{2}g_{ij}v_k v^k, \tag{4}$$

together with the wave equation

$$g^{ij}v_{;ij} = \sigma, \tag{5}$$

where the micro matter scalar field v and the source density σ are both functions of cosmic time t only. The semicolon (;) denotes covariant derivative with respect to g_{ij}.

Using (4), the field equations (2) and (3) for the metric (1) can be written as

$$2\beta_{44} + 3\beta_4^2 = -4\pi\phi^{-1}v_4^2, \tag{6}$$

$$\alpha_{44} + \beta_{44} + \alpha_4\beta_4 + \alpha_4^2 + \beta_4^2 = -4\pi\phi^{-1}v_4^2, \tag{7}$$

$$2\alpha_4\beta_4 + \beta_4^2 = 4\pi\phi^{-1}v_4^2, \tag{8}$$

$$\phi_{44} + (\alpha_4 + 2\beta_4)\phi_4 = -\frac{8\pi}{3}\lambda v_4^2, \tag{9}$$

where suffix 4 denotes ordinary differentiation with respect to time t.

From (6), (7) and (8), we obtain

$$\begin{aligned} \alpha_{44} + 2\beta_{44} + 4\alpha_4\beta_4 + \alpha_4^2 + 4\beta_4^2 &= 0, \\ \frac{(e^\alpha e^{2\beta})_{44}}{e^\alpha e^{2\beta}} &= 0, \end{aligned} \tag{10}$$

which is equivalent to

$$(e^\alpha e^{2\beta})_{44} = 0. \tag{11}$$

On integration, we get

$$\begin{aligned} e^{\alpha}e^{2\beta} &= n_1 t + n_2, \quad \text{where } n_1 \neq 0,\ n_1 \text{ and } n_2 \text{ are arbitrary constants of integration,} \\ e^{\alpha}e^{2\beta} &= T, \quad \text{where } T = n_1 t + n_2 \text{ (here } n_1 = 1,\ n_2 = 0) \end{aligned} \tag{12}$$

(i.e. transforming the time co-ordinate t by putting $n_1 t + n_2 = T$).

From (12) we can write the explicit form for e^{α} and e^{β} as

$$\begin{aligned} e^{\alpha} &= T^{k_1} \quad \text{and} \quad e^{\beta} = T^{k_2}, \\ e^{\alpha}e^{2\beta} &= T^{k_1+2k_2}, \\ e^{\alpha}e^{2\beta} &= T \quad \text{satisfying the relation } k_1 + 2k_2 = 1. \end{aligned} \tag{13}$$

Equation (13) will act as a particular solution of above field equations.

From (8), we get

$$v_4^2 = v_T^2 = \frac{[k_2^2 + 2k_1k_2]\phi}{4\pi T^2} \quad \text{(where } n_1 = 1,\ n_2 = 0 \text{ and } t \to 4). \tag{14}$$

Hence (9) will reduce to

$$\phi_{TT} + \frac{1}{T}\phi_T + \frac{8\pi}{3}\lambda v_T^2 = 0. \tag{15}$$

From (14) and (15), we obtain

$$T^2\phi_{TT} + T\phi_T + \omega^2\phi = 0, \tag{16}$$

where $\omega = \sqrt{\frac{2\lambda}{3}(k_2^2 + 2k_1k_2)}$ and $0 < \lambda < 10^{-1}$ with $k_2^2 + 2k_1k_2 > 0$. On integration, (16) yields two basic solutions for ϕ i.e.

$$\phi_1 = \cos(\omega \log T) \quad \text{and} \tag{17}$$

$$\phi_2 = \sin(\omega \log T). \tag{18}$$

The value ϕ_2 (second value of ϕ given in (18)) is not of much importance as it leads to the unphysical situation.

From (14), we get

$$v = r_1 \int \frac{\sqrt{\phi}}{T} dT + r_2, \tag{19}$$

where r_2 is the constant of integration, and

$$r_1 = \sqrt{\frac{k_2^2 + 2k_1k_2}{4\pi}}. \tag{20}$$

Using (17) and (18) in (19), we obtain the expression for micro matter scalar field v as

$$v = r_1 \int \frac{\sqrt{\cos(\omega \log T)}}{T} dT + r_2 \tag{21}$$

and

$$v = r_1 \int \frac{\sqrt{\sin(\omega \log T)}}{T} dT + \alpha_2, \tag{22}$$

but $\int \sqrt{\cos x}\, dx = 2E(x/4)$, where $E(x/m)$ represents Elliptic integral of second kind.

Expansion of $E(x/m)$ in power series is

$$E(x/m) = x - \frac{mx^3}{6} - \frac{m(3m-4)}{120}x^5 - \frac{(m(16 - 60m + 45m2)x^7}{5040} - \cdots. \tag{23}$$

Using (23), equivalent form of (21) are

$$v = r_1 \left[\log T - \frac{\omega^2}{12}(\log T)^3 - \frac{\omega^4}{480}(\log T)^5 - \cdots \right] + r_2, \tag{24}$$

similarly

$$v = r_1 \left[\frac{2}{3}(\omega)^{1/2}(\log T)^{3/2} - (\omega)^{5/2}\frac{(\log T)^{7/2}}{42} + \cdots \right] + r_2. \tag{25}$$

(N.B.: The expansion of the solution given in (24) and (25) is only valid for small T.)

In new coordinate system, the source density σ of the micro matter scalar field v given by (5) for the space time (1) reduces to

$$\sigma = -\left[v_{TT} + \frac{v_T}{T} \right]. \tag{26}$$

With the help of (21) and (22), (26) reduces to

$$\sigma = \frac{r_1 \omega}{2T^2} \frac{\sin(\omega \log T)}{\sqrt{\cos(\omega \log T)}}, \tag{27}$$

$$\sigma = -\frac{r_1 \omega}{2T^2} \frac{\cos(\omega \log T)}{\sqrt{\sin(\omega \log T)}}. \tag{28}$$

The energy density ρ associated with the micro matter scalar field v [18, 19] is given by

$$\rho = \frac{1}{2} v_4^2 = (v_T)^2/2. \tag{29}$$

Using (14) and (20), we get

$$\rho = \frac{1}{2T^2} r_1^2 \phi,$$

$$\rho = \frac{r_1^2}{2T^2}[\cos(\omega \log T)] \quad \text{and} \tag{30}$$

$$\rho = \frac{r_1^2}{2T^2}[\sin(\omega \log T)]. \tag{31}$$

The axially symmetric Bianchi type-I cosmological model in Barber's second self creation theory with micro matter scalar field will be given by

$$ds^2 = -dT^2 + T^{2k_1} dX^2 + T^{2k_2}(dY^2 + dZ^2), \quad \text{where } k_1 + 2k_2 = 1. \tag{32}$$

3 Some Physical and Kinematical Properties of the Model

$$\text{(i)} \quad \text{Spatial volume } = \sqrt{-g} = T. \tag{33}$$

$$\text{(ii)} \quad \text{Scalar expansion } \theta = u^i_{;j} = \frac{k_1}{T} + \frac{2k_2}{T} = 1/T. \tag{34}$$

$$\text{(iii)} \quad \text{Shear scalar } \sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{7}{18T^2}. \tag{35}$$

$$\text{(iv)} \quad \lim_{T\to\infty}\left(\frac{\sigma}{\theta}\right) \neq 0. \tag{36}$$

The Barber's scalar field ϕ, the micro matter scalar field ν, the source density σ and the energy density ρ are functions of time T and are given by (17), (24), (27) and (30) respectively.

(a) When $T = 1$, then we get $\phi = 1$, $\nu =$ constant, $\rho = 0$ or constant and $\sigma = 0$. In this case the space time reduces to a flat space time.
(b) When $T \to 0$ or ∞, then the quantities ϕ, ν, ρ and σ are undetermined. The metric potentials e^{α}, $e^{2\beta}$ i.e. $e^{\alpha+2\beta}$ tend to zero as $T \to 0$. Therefore, the space time collapses at $T = 0$ and admits a singularity at $T = \infty$.
(c) When the coupling parameter $\lambda \to 0$ ($T \neq 0$ or ∞) then $\phi \to 1$, $\nu \to$ constant, $\sigma \to 0$ and $\rho \to$ constant. It clearly indicates that Barber's theory leads to Einstein theory as $\lambda \to 0$.
(d) The spatial volume $\to 0$ as $T \to 0$ and volume $\to \pm\infty$ as $T \to \pm\infty$. Which implies that the universe starts expanding with zero volume and blows up at infinite past and future.
(e) One can observe that scalar expansion $\theta \to 0$ as $T \to \infty$ and $\theta \to \infty$ as $T \to 0$. Thus the universe is expanding with increase of time but the rate of expansion becomes slow as time increases (where $k_1 + 2k_2 = 1$).
(f) Shear scalar $\sigma^2 \to 0$ as $T \to \infty$ and $\sigma^2 \to \infty$ as $T \to 0$. Thus the shape of the universe changes uniformly.
(g) It is observed that $\lim_{T\to\infty}(\frac{\sigma}{\theta}) \neq 0$ which confirms that the universe remains anisotropic throughout the evolution.

4 Conclusion

Here we have constructed the anisotropic homogeneous axially symmetric Bianchi type-I micro model in Barber's second self creation theory. It is interesting to note that Barber's scalar $\phi \to 1$ as the coupling constant $\lambda \to 0$ ($T \neq 0$ or ∞). Also when $\lambda \to 0$ we get $\omega \to 0$. Consequently the massless scalar field ν and the source density σ in Barber's theory will tend to the corresponding quantities in Einstein theory. Hence, Barber's theory is valid in all respects and leads to Einstein theory as $\lambda \to 0$ in micro level. Therefore, the model represented by (32) has importance in self creation cosmology for the study of dynamics in quantum level.

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